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DUAL AUTOMORPHISM-INVARIANT MODULES OVER PERFECT RINGS

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Abstract: Under study are the dual automorphism-invariant modules and pseudoprojective modules. Some conditions were found under which the dual automorphism-invariant module over a perfect ring is quasiprojective. We also show that if R is a right perfect ring then a pseudoprojective right R-module M is finitely generated if and only if M is a Hopf module.

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1. Introduction

All rings are assumed associative and unitary, and the modules are unitary as well. A submodule N of a module M is *small* in M provided that $N + N' \neq M$ for every proper submodule N' of M. We denote the fact that N is a (small) submodule of M by $N \leq M$ (respectively, by $N \ll M$). A module M is *dual automorphism-invariant* if, for all small submodules K_1 and K_2 of M, every epimorphism $f : M/K_1 \to M/K_2$ with $\mathrm{Ker}(f) \ll M/K_1$ can be lifted to a homomorphism $f' : M \to M$. The dual automorphism-invariant modules were firstly studied in [1]. A module M is *quasiprojective* (*pseudoprojective*) provided that for every submodule N of M, each homomorphism (epimorphism) $f : M \to M/N$ can be lifted to a homomorphism $f' : M \to M$. By [1, Proposition 7] every pseudoprojective module is dual automorphism-invariant. The converse holds for the right modules over right perfect rings (see [2]). Recently the dual automorphism-invariant modules and their analogs have been studied in [1–7].

In this article we consider the dual automorphism-invariant and pseudoprojective modules. The main results are connected with the properties of the dual automorphism-invariant modules over perfect rings. In Section 2 we consider conditions for a dual automorphism-invariant module over a perfect ring to be quasiprojective. Also, we establish that every right module M over a right perfect ring is dual automorphism-invariant if and only if, for every submodule N of M, each automorphism of M/N can be lifted to an automorphism of M. In Section 3 we show that if R is a right perfect ring then every pseudoprojective right R-module M is finitely generated if and only if M is a Hopf module.

We denote by $J(R)$ and $J(M)$ the Jacobson radical of a ring R and a module M. The fact that right R-modules M and N are isomorphic will be denoted by $M \cong N$. We let $\mathrm{End}(M)$ and $\mathrm{Aut}(M)$ stand for the endomorphism and automorphism rings of a right R-module M.

In this article we use the standard notions of ring theory (for example, see [8–10]).

2. Dual Automorphism-Invariant Modules

An epimorphism $f : P \to M$ of right R-modules is a *projective cover* of M provided that P is a projective module and $\mathrm{Ker}(f) \ll P$.

Lemma 1. *Let $M_1 \oplus M_2$ be some dual automorphism-invariant modules, and let $p_1 : P_1 \to M_1$ and $p_2 : P_2 \to M_2$ be their projective covers. If $P_1 \cong P_2$ then $M_1 \cong M_2$.*

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